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A QUEUE MODEL FOR MOTOR VEHICLES DISMANTLING AND RECYCLING IN A SCARCE ENERGY AMBIENCE

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Abstract. We propose the M|G|∞ queue system to study a situation in which conventional motor vehicles will be out of use as far as the usual energy sources will be exhausted. Two possibilities are posed: recycling or dismantling. Using the hazard rate function we conclude that when the rate of dismantling and recycling of motor vehicles is greater than the rate at which vehicles become idle, the system tends to get balanced. The motor vehicles that become unused with the conventional energy turn useful with another kind of energy or are included in other useful devices.

Key words. Motor Vehicles, Recycling, Dismantling, M|G|∞, Hazard Rate Function.

Mathematics Subject Classification: Primary 60E05, 60G07; Secondary 60K25.

1 Introduction

Hardin's paper of "The tragedy of the commons" [7] is a reference for the problems that traditionally occur in the resources area. Theoretical study of Commons became extremely important in the analysis of the consequences of human behavior when human specie exploits Earth resources [1,6]. Scientists have long claimed for the need of changing human behavior and we are arriving at the last chance to change things. Some apocalyptic studies point out this evidence very clearly [9].

New situations may occur very suddenly. The production of oil has already got its peak and new oil productions will occur in the future with decreasing rents until the complete depletion of this resource. The other non-renewable sources of energy will have the same end.

We may have to reorder the priorities and to reorganize structures in societies. We have now to produce the new kind of energies (clean energies) at a major scale. The big problem that remains is to know if the transition period from non-renewable sources of energy to the renewable sources is enough to overcome the big problems related with the destruction of Earth. All the wastes that people have made for so many decades must be overcome, as well. Anyway, many kinds of new problems will occur. However, what is important now is to know how quickly changes may happen while we develop the new sources of energy in order to create a new economy and a reorganized society.

Two situations may occur. First, it is possible to reverse climatic changes and to reverse all the related problems with a fast change of direction from the old to the new sources of energy. Anyway, many types of equipment must be recycled to respond to the new situation as far as factories begin to produce new equipments responding to the new circumstances.

The other situation represents a tragic scenario where there is no enough time to perform a smooth transition and societies would have a period of big privations [7]. In our opinion, such a scenario will not be the prevailing one if institutions realize about the big problems of all kinds for our global civilization.

2 A queue model for motor vehicles' dismantling and recycling

We consider the M|G|∞ queue system [8] where customers arrive according to a Poisson process at rate λ [4]. They receive a service whose length is a positive random variable with distribution function G(.) and mean α. Each customer as soon as it arrives at the system, immediately finds an available server. Each customer service is independent from the others customers' services and from the arrival process. The traffic intensity is given by $\rho = \lambda\alpha$.

With this model we intend to analyze a situation in which motor vehicles arrive at the system getting idle and leave the system as soon as they are recycled or dismantled. Both situations are modeled with the same purpose in the model. Our interest in studying this situation is precisely to see how the system may recover to a balanced situation in which motor vehicles get operational or get dismantled (in this situation materials would become employed as components in other applications).

Let $N(t)$ be the number of busy servers (or, what is the same, the number of customers being served) in the instant t, in a M|G|∞ system. If we consider $p_{0n}(t) = P[N(t) = n | N(0) = 0]$, $n = 0,1,2,...$, we may have [2]:

$$p_{0n}(t) = \frac{\left(\lambda \int_0^t [1 - G(v)]dv\right)^n}{n!} e^{-\lambda \int_0^t [1-G(v)]dv}, \quad n = 0,1,2,... \tag{1}$$

So if the initial instant is a moment at which the system is empty, the transient distribution is Poisson with mean $\lambda \int_0^t [1 - G(v)]dv$.

The stationary distribution is the limit one:

$$\lim_{t\to\infty} p_{0n}(t) = \frac{\rho^n}{n!} e^{-\rho}, \quad n = 1,2,... \tag{2}$$

This queue system, as any other, has a sequence of busy periods and idle periods. A busy period begins when a customer arrives at the system, finding it empty.

Let's see the distribution of the number of customers being served in the instant t in the M|G|∞ system, when the initial instant is the moment at which a busy period begins, that gets relevant for our purposes.

Be $p_{1'n} = P[N(t) = n | N(0) = 1']$, $n = 0,1,2,...,$ and $N(0) = 1'$ the initial instant at which a customer arrives at the system and the number of customers being served turns from 0 to 1. This means that a busy period has just begun [3].

So at the moment $t \geq 0$, we may have a situation that represents [5]:

1. the customer that arrived at the system at the initial instant has left the system with a probability $G(t)$, or he remains in the system, with probability $1 - G(t)$;
2. the other servers, which were empty at the beginning (initial instant), may be now empty or busy with 1, 2, ... customers, with probabilities given by $p_{0n}(t)$, $n = 0,1,2,...$.

The two subsystems, the one of the initial customer and the one of servers initially empty, are independent. Consequently:

$$p_{1'0}(t) = p_{00}(t) G(t)$$

$$p_{1'n}(t) = p_{0n}(t) G(t) + p_{0n-1}(t)(1 - G(t)), \qquad n = 1, 2, \ldots \tag{3}$$

We also have for this situation:

$$\lim_{t \to \infty} p_{1'n}(t) = \frac{\rho^n}{n!} e^{-\rho}, \quad n = 0, 1, 2, \ldots \tag{4}$$

For the M|M|∞ system (exponential service times), the equations 3 are applicable even when $N(0) = 1$ (when the initial instant is a moment at which there is a customer in the system; that does not enforce that this is the moment at which we are turning from 0 to 1 customer to be served). This results from the lack of memory of the exponential distribution.

If $g(t)$ is the probability density function related to $G(t)$, and if we call $h(t)$ the hazard rate function, we'll have [10]:

$$h(t) = \frac{g(t)}{1 - G(t)} \tag{5}$$

The function $h(t)$ is the rate at which services end.

So,

Proposition 1:

If $G(t) < 1$, $t > 0$, continuous and differentiable and if

$$h(t) \geq \lambda G(t), \; t > 0 \tag{6}$$

$p_{1'0}(t)$ is a non-decreasing function.

Dem:

It's enough to observe that

$$\frac{d}{dt} p_{1'0}(t) = p_{00}(t)(1 - G(t)) \left(\frac{g(t)}{1 - G(t)} - \lambda G(t) \right).$$

Besides, we may note that

$$h(t) \geq \lambda, \quad t > 0 \tag{7}$$

is a sufficient condition for the result in 6.

So, if the rate at which the services end is greater or equal than the rate of arrivals, we conclude that $p_{1'0}(t)$ does not decrease.

For the system M|M|∞, the equation 7 is equivalent to

$$\rho \ \leq \ 1 \tag{8}$$

Considering $\mu(1',t)$ and $\mu(0,t)$ the mean values of the distributions given by 3 and 1, respectively, we'll have

$$\mu(1',t)=\sum_{n=1}^{\infty} np_{1'n}(t)=\sum_{n=1}^{\infty} nG(t)\,p_{00}(t)+\sum_{n=1}^{\infty} np_{0n-1}(t)\big(1-G(t)\big)=$$
$$=G(t)\mu(0,t)+\big(1-G(t)\big)\sum_{j=0}^{\infty}(j+1)p_{0j}(t)=\mu(0,t)+\big(1-G(t)\big).$$

So,

$$\mu(1',t)=1-G(t)+\lambda\int_0^t\big[1-G(v)\big]dv \tag{9}$$

Proposition 2

If $G(t)<1,\ \ t>0$, continuous and differentiable and if

$$h(t)\leq\lambda,\quad t>0 \tag{10}$$

$\mu(1',t)$ is a non-decreasing function.

Dem:

It's enough to observe that, considering equation 9, $\frac{d}{dt}\mu(1',t)=\big(1-G(t)\big)\big(\lambda-h(t)\big)$.

Besides, if the rate at which services end is lesser or equal than the rate at which customers arrive $\mu(1',t)$ is a non-decreasing function. We can note additionally that, for the M|M|∞ system, the equation 10 is equivalent to

$$\rho \ \geq \ 1 \tag{11}$$

3 Results and comments

According to our study interests, the customers are the motor vehicles that become idle. The arrival rate is the rate at which the motor vehicles become idle. The service time for each one is the time that goes from the instant they get idle until the instant they become recycled or dismantled. The service time hazard rate function is the rate at which the motor vehicles become recycled or dismantled.

An idle period for our M|G|∞ system should be a one at which there were no motor vehicles idle. In a busy period there are always continuously idle motor vehicles.

The equation 6 shows that if the dismantling and recycling rate is greater or equal than the rate at which motor vehicles get idle, the probability that the system gets empty (that is, there are no idle motor vehicles) does not decrease with time. This means that the system has a tendency to become balanced as far as time goes on.

The equation 10 shows that if the dismantling and recycling rate is lesser or equal than the rate at which motor vehicles get idle, the mean number of motor vehicles in the system does not decrease with time. This means that the system has a tendency to become unbalanced as far as time goes on.

Consequently, we conclude that when the rate of dismantling and recycling of motor vehicles is greater than the rate at which they become idle, the system has a tendency to get balanced. In this situation, the motor vehicles that become unused with the conventional energy turn useful with another kind of energy or get included in other useful devices.

We must note that it is important the recycling or the dismantling of motor vehicles but, more than that, it is essentially relevant the cadence at which these actions are performed. Moreover, we give a reference for this cadence: λ, the rate at which motor vehicles get idle.

4 An economic analysis as a complement to the model

We have seen that rates λ and *h(t)* are determinant to monitor the way that the system of motor vehicles recycling and dismantling may be managed.

We consider now additionally p as the probability for the motor vehicles arrivals destined to recycling and (1-p) as the probability for the motor vehicles arrivals destined to dismantling. Let $h_i(t)$, $c_i(t)$ and $b_i(t)$, $i=1,2$ be the hazard rate function, the mean cost and the mean benefit, respectively for recycling when $i=1$ and dismantling when $i=2$.

With these new variables we can perform an economic analysis (beyond other considerations that may be posed) to evaluate about the interest of recycling and dismantling.

We will analyze this situation in a global approach and not in a selfish way, considering the individuals point of view.

So, we may consider the total cost per unit of time for motor vehicles recycling and dismantling as:

$$C(t) = pc_1(t)\lambda + (1-p)c_2(t)\lambda \qquad (12)$$

Furthermore, the benefit per unit of time resulting from recycling and dismantling is given by:

$$B(t) = b_1(t)h_1(t) + b_2(t)h_2(t) \qquad (13)$$

From an economic point of view, it must be B(t)>C(t).
To conclude about the advantage of recycling, we have the following:

$$b_1(t) > \max\left[\ \frac{p\lambda c_1(t) + (1-p)\lambda c_2(t) - b_2(t)h_2(t)}{h_1(t)}, 0\ \right] \qquad (14)$$

With $G_1(t)$ and $G_2(t)$ both exponential, equation 14 becomes:

$$b_1(t) > \max\left[\ (pc_1(t) + (1-p)c_2(t))\rho_1 - \frac{\alpha_1}{\alpha_2} b_2(t), 0\ \right] \qquad (15)$$

To conclude about the advantage of dismantling we have, in the same conditions:

$$b_2(t) > \max\left[\ \frac{p\lambda c_1(t) + (1-p)\lambda c_2(t) - b_1(t)h_1(t)}{h_2(t)}, 0\ \right] \qquad (16)$$

and

$$b_2(t) > \max\left[\ (pc_1(t) + (1-p)c_2(t))\rho_2 - \frac{\alpha_2}{\alpha_1} b_1(t), 0\ \right] \tag{17}$$

So, there are minimum benefits above which, from an economic point of view both, recycling and dismantling, are interesting. The most interesting is the one for which this minimum benefit is the least. By other words: in a global perspective, it is more efficient the activity that corresponds to a lower level for the minimum interesting benefit.

Recycling seems to be as much interesting as far as it is more economically profitable and our inequalities 14 to 17 are tools that may be applied to evaluate this interest.

Note that instead of the instantaneous reasoning we can make it for a period of time of length T. It's enough to perform the respective integral calculus between 0 and T. Of course, the conclusions are analogous.

5 Main conclusions

Our model contributes for a better understanding of this kind of problems and it (or some modified versions of it) may be applied to study some other social and economic phenomena such as unemployment, health or projects of investment, for example, with interesting results.

An extension of our model has also permitted to get conclusions about the economic advantages of recycling and dismantling.

The application of the model to the phenomenon studied in our paper shows that our model is very useful and that its conclusions and results are quite simple to understand. Just through a theoretical analysis of the model, we have evidenced some remarkable topics for analyzing the evolution of the studied system (or another one, whichever it is, since it is according the assumptions of the model).

In practice, it is essential to estimate λ and $h(t)$ to get conclusive particular results for the available data about the system. This will give us the tools to monitor the situation and to suggest solutions. A correct estimation of λ will depend on the arrivals process to be Poisson, in real. Additionally, in general, it is correct to admit that with very large populations, such as the one we are dealing with, the estimation of $h(t)$ is usually technically complicated. So, frequently, the best to do is to estimate directly $h(t)$ instead of estimating first the service time distribution and then computing $h(t)$.

A particular situation at which the computation is easier is the exponential service time one, for which $h(t) = 1/\alpha$.

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